Lecture 2 Conservation laws and planetary waves

- Potential vorticity
- Orographic forcing of Rossby waves
- Barotropic Instability
- Baroclinic Instability
- Fronts and wave breaking
- Midlatitude storm tracks
Planetary Balance and conservation

If the time scale $\tau$ of the motion is larger than:

$$\tau > f^{-1} > N^{-1}$$

$$\tau > 20\text{hs} > 10\text{min}$$

And for scales large such $U < L/f$, $L<\sim a$ (earth radius).

The flow is in *hydrostatic balance*

Buoyancy $\sim$ vertical component of pressure force. And in *geostrophic balance*:

- coriolis force $\sim$ horizontal component of pressure force.
Hydrostatic Balance

\[
\frac{\partial P}{\partial z} = -\rho g
\]

\[
P = \left( \frac{P}{P_0} \right)^\kappa
\]

\[
\frac{1}{\rho} \frac{\partial P}{\partial z} = C_p \theta \frac{\partial \pi}{\partial z}
\]

Geostrophic Balance

If the Rossby Number is much smaller than 1 \((R_0 = U/(fL) \ll 1)\)

\[
-u2\Omega \sin \phi_0 = \frac{C_p \theta_0}{a \cos \phi_0} \frac{\partial \pi}{\partial \lambda}
\]

\[
v2\Omega \sin \phi_0 = \frac{C_p \theta_0}{a} \frac{\partial \pi}{\partial \phi}
\]

Where \(u\) and \(v\) are zonal and meridional velocity components and \(\phi_0\) a particular latitude
Conservation of Potential Vorticity

**Full system**

\[
\frac{\partial Q}{\partial t} + u \frac{\partial Q}{\partial x} + v \frac{\partial Q}{\partial y} + w \frac{\partial Q}{\partial z} = 0
\]

\[Q = (f + \zeta) \theta_z + \eta \theta_y + \mu \theta_x\]

\[\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\]

\[\eta = \frac{\partial u}{\partial z} - \frac{\partial w}{\partial x}\]

\[\mu = \frac{\partial w}{\partial x} - \frac{\partial v}{\partial z}\]

**Quasi-geostrophic**

\[
\frac{\partial q}{\partial t} + u_g \frac{\partial q}{\partial x} + v_g \frac{\partial q}{\partial y} = 0
\]

\[q = (\zeta + f_0 + \beta y + \frac{f_0}{\rho_r} \frac{\partial}{\partial z} \left\{ \frac{\rho_r}{N^2} \phi_z \right\}\]

\[\zeta = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} = \nabla^2 \phi\]

**Shallow water**

\[
\frac{\partial q_s}{\partial t} + u_s \frac{\partial q_s}{\partial x} + v_s \frac{\partial q_s}{\partial y} = 0
\]

\[q_s = \left( \frac{\zeta + f}{H + h} \right)\]

\[N^2 = \frac{g}{\theta_r} \frac{\partial \theta}{\partial z}\]
Rossby Waves

Phase group (Energy flows to the east)

Possibility of stationary because:

\[ Q_s = \frac{\zeta_2 + f_2}{H} \]

\[ Q_s = \frac{\zeta_1 + f_1}{H} \]

\[ \zeta_2 > \zeta_1 \]
Fig. 4.7  A cylindrical column of air moving adiabatically, conserving potential vorticity.

3 Named for the German meteorologist Hans Ertel. A more general form of Ertel's potential vorticity is discussed, for example, in Gill (1982). Potential vorticity is often expressed in the potential vorticity unit (PVU), where 1 PVU = $10^{-6}$ K kg$^{-1}$ m$^2$ s$^{-1}$. 
Stationary response due to orography (shallow water global model)

U zonal-comp.

V meridional comp.

H height

PV potential vorticity
Diagnostics from spectral shallow water model

DATA SET: shallow

PV – PV[X = @AVE]
Figure 6. The quasigeostrophic solution for steady flow over a semi-infinite ridge in a semi-infinite atmosphere. The plots show (top) total buoyancy (solid) and perturbation (dashed) buoyancy, (middle) meridional velocity perturbation, and (bottom) relative vorticity perturbation. The heavy contour indicates the mountain profile.
\( U_0 > 0 \)
Westerly flow

\( U_0 < 0 \)
Easterly flow
Response to large-scale orography
Barotropic and Baroclinic Instabilities

The major source of midlatitudes weather
Diagnostics from spectral shallow water model

DATA SET: shallow

geopotential (m²/s²)
Diagnostics from spectral shallow water model

LONGITUDE: 180E
LATITUDE: 38.5N
DATA SET: shallow

$u_{wind} \text{ (m/s)}$

$m/s$

T
LONGITUDE: 0.7W(-0.7) to 0.7W(359.3)
T: 0.5 to 50.5

Diagnostics from spectral shallow water model

PV[X=@AVE,L=1.50@AVE]/PV[X=@AVE,L=1.50@AVE,Y=@MAX]
Zonal Mean Flow and Potential Temperature for the Baroclinic Simulations
Thermal wind relation
\[ f^* U_z = -g^* \theta_y / \Theta \]

The phase velocity:
\[ C = U(z=0) + U_z(z=0) / \gamma \]

The perturbation temperature
\[ q = q_0 \exp \left[ ik(x-ct) \right] \exp(-\gamma z) \]
Fig. 8.1  A schematic picture of cyclogenesis associated with the arrival of an upper-level positive vorticity perturbation over a lower-level baroclinic region. (a) Lower-level cyclonic vorticity induced by the upper-level vorticity anomaly. The circulation induced by the vorticity anomaly is shown by the solid arrows, and potential temperature contours are shown at the lower boundary. The advection of potential temperature by the induced lower-level circulation leads to a warm anomaly slightly east of the upper-level vorticity anomaly. This in turn will induce a cyclonic circulation as shown by the open arrows in (b). The induced upper-level circulation will reinforce the original upper-level anomaly and can lead to amplification of the disturbance. (After Hoskins et al., 1985.)
LATITUDE : 44.6N
TIME : 04–JAN–1900 12:00

DATA SET: EADY_jet003.b1.e9601.nc
/archive/io/EADY_jet003.b1.e9601.nc

TH–TH[T=0]
Fig. 10.13  The observed mean energy cycle for the Northern Hemisphere. Numbers in squares are energy amounts in units of $10^5 \text{ J m}^{-2}$. Numbers next to arrows are energy transformation rates in units of $\text{W m}^{-2}$. $B(\overline{P})$ represents a net energy flux into the Southern Hemisphere. Other symbols are defined in the text. (Adapted from Oort and Peixoto, 1974.)
Baroclinic effects:

Atmospheric Fronts

Transfer of momentum and heat poleward

Breaking upper and lower waves
Potential Vorticity of the Shallow Water Global Model
for three different amplitudes of the forcing

\[ F(\lambda, \phi, t) = W \left( \frac{U(\phi) - c}{U_{\text{max}}} \right) G(t) \cos(m\lambda - ct) \]

Where \( U(\phi) \) is the initial jet, \( G(t) = e^{-(t-t_0)/t_1} \) grows and decays in 8 days, \( C = 0.6 \) \( U_{\text{max}} \) \( U_{\text{max}} = 40m/s \), in the local wave number this case is \( m = 7 \).

**Weak Anticyclonic Wavebreaking**
Equatorial Rossby wave propagation absorbed by the critical layer due to the mean flow.

**Strong Anticyclonic Wavebreaking**
For intense vortices, if the anticyclone is strong enough it can pull and shed the cyclonic centers.

**Strong Cyclonic Wavebreaking**
However, if the cyclone centers are very strong they can pull and shed the anticyclonic vorticity.
Weak Anticyclonic Wave Breaking

Critical Layer $U = C$
Collective effects of baroclinic waves (daily weather)

Produce

*Storm tracks* and quasi stationary patterns affecting the basic flow (westerlies)
Observed Precipitation $V_{925}$ and $Z_{500}$


Mark Rodwell
Storm Track (~ season)

Cyclone System (~one week)

Frontal System (~2 days)

Cloud System (~2 hours)

Kilometers in length

Kilometers in width

Storm Track (~ season)

Cyclone System (~one week)

Frontal System (~2 days)

Cloud System (~2 hours)

Time-space scale of atmospheric systems

Non-hydrostatic dynamics

Hydrostatic dynamics
The mechanics of a storm track and its feedback

Environment for cyclone development.

Produce frontal systems and low level convergence

Cyclone system

Intense cyclone waves can modify the storm track

Frontal system

Intense fronts produce more cyclogenesis

Cloud system

The cloud System releases latent heat that further intensifies the front

Strong vertical ascents, associated with the front, transport upward moisture and other tracers

Well resolved by present GCM’s
Fairly resolved
Only parameterized
Fig. 10.16 Meridional cross sections showing the relationship between the time mean secondary meridional circulation (continuous thin lines with arrows) and the jet stream (denoted by J) at locations (a) upstream and (b) downstream from the jet stream cores. (After Blackmon et al., 1977. Reproduced with permission of the American Meteorological Society.)
Surfaces of relative vorticity as inferred from regression analysis (-24h, 0h, +24h)

I. Orlanski and B. Gross: Baroclinic lifecycles in a storm track environment. JAS 2000
Mean intensity of the cyclones for DJF: 1979-95

Patterns quite well simulated, although peak intensity generally underestimated in HadAM3. Resolution?
Pacific storm track is too strong. Atlantic storm track too weak in HadAM3, possibly related to lack of systems coming off the eastern side of the Rockies.
Number of Cyclones DJFM 1982-2001
Experiments on Storm Track variability

A high resolution (9km and 18km) cloud resolving non-hydrostatic model (ZETANC*) was used to perform several idealized storm track simulations. The solution run for 220 days and sensitivity to imposed SST were performed. The animation shows the column liquid water content.

The frontal rain band ahead of cold fronts displays a variety of cloud systems, from deep convective clouds in sub tropical latitudes to more stable stratiform clouds in the middle latitudes.
Things to remember from Lecture 2.

- Planetary waves are an essential part of the daily weather and climate.
- Land-sea contrast, topographic features etc can force planetary waves, Rossby waves.
- Its characteristics are; they propagate to the west and radiate energy to the east.
- Also these waves could be unstable, they can grow to large amplitudes from very small perturbations.
- The major planetary wave generation are due to barotropic and baroclinic instabilities. They are responsible for producing the daily weather. Most the time they transport heat and momentum to higher latitudes. Warming the polar regions and removing heat from the subtropics.
- The assemble of all these waves tend to produce wave active regions which we called the storm tracks.